Analysis of helical armature windings with particular reference to superconducting a.c. generators


Indexing terms: Generators, Superconductivity

Abstract: An unusual form of polylphase armature winding is described in which conductors lie on a helix of constant radius as they pass from one end of the machine to the other. There are no end windings. An analysis of the winding is performed which allows analytical expressions for the flux densities and machine reactances to be found. Measured and calculated values of synchronous reactance agree to within 2%. Some of the more unusual properties of helical windings are discussed.

List of symbols

\[ H = \text{magnetic field vector, A/m} \]
\[ H_r = \text{radial component of magnetic field, A/m} \]
\[ H_a = \text{axial component of magnetic field, A/m} \]
\[ H_b = \text{tangential component of magnetic field, A/m} \]
\[ I = \text{peak phase current, A} \]
\[ I_{0}(nkr) = \text{modified Bessel function of first kind, order } n, \text{ argument } nkr \]
\[ I_{0}^{*}(nkr) = (d/dn)I_{0}(nkr) \]
\[ K_{0}(nkr) = \text{modified Bessel function of second kind, order } n, \text{ argument } nkr \]
\[ K_{0}^{*}(nkr) = (d/dn)K_{0}(nkr) \]
\[ K = \text{tangential component of linear current density, A/m} \]
\[ K_b = \text{axial component of linear current density, A/m} \]
\[ L = \text{self-inductance, H} \]
\[ M = \text{mutual inductance, H} \]
\[ N_{ph} = \text{number of turns in series per phase} = Z/2a \]
\[ Z = \text{number of armature conductors per phase} \]
\[ A_{91}, A_{92}, A_{93}, A_{94}, A_{95}, A_{96}, A_{97}, A_{98} = \text{Constants in general solution of Laplace's equation} \]
\[ \Phi_{1}, \Phi_{2}, \Phi_{3}, \Phi_{4}, \Phi_{5}, \Phi_{6} = \text{magnetic flux linkage} \]
\[ \Omega = \text{magnetic scalar potential} \]
\[ \beta_{0}(nkr) = K_{0}(nkr)/I_{0}(nkr) \]
\[ \gamma_{0}(nkr) = K_{0}(nkr)/I_{0}^{*}(nkr) \]
\[ a = \text{number of parallel paths per phase} \]
\[ b = \text{number of phasebands per phase} \]
\[ f = \text{system frequency (50 Hz)} \]
\[ k_{un} = \text{nth harmonic 1000 rev/min turbo-generator} \]
\[ k_{bn} = \text{nth harmonic winding factor} = k_{0}k_{m} \]
\[ k_{bn} = \text{nth harmonic breadth factor} \]
\[ k_{rn} = \text{nth harmonic radial-flux-density environmental screen factor} \]
\[ k_{en} = \text{nth harmonic tangential-flux-density environmental screen factor} \]

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1 Introduction

Since 1940 the unit size of two-pole 3000 rev/min turbo-generators has increased from 60 to 660 MW. At the same time, the specific power output has increased from 0.3 kW/kg to 2 kW/kg. In order words, there has been almost a sevenfold improvement in the power/weight ratio. This has been made possible by the introduction of hydrogen cooling and, more recently, water cooling, of the stator conductors, which has enabled more effective use to be made of the iron and copper within the machine.

Within the framework of the maximum magnetic and
electric loadings used today, it appears there may be
difficulties in increasing the size of conventional generators
beyond 2000 MW. Difficulties which arise because of two
limits. These are: first, the limit on maximum indivisible
weight which is set by power station craneage and bridge
and ship loadings; secondly, the limit on the length between
rotor bearings, which is set by the critical speeds. One
possible way of overcoming weight limitations would be
by subdividing the inner stator, which is usually the heaviest
item, and building it on site from smaller factory-built
assemblies. Other ways of overcoming, for instance, the
problem of rotor length, are by developing slotless
windings or alternatively, superconducting machines.
In both cases, higher specific power outputs become
possible, albeit at the cost of introducing new technology.
One advantage of either slotless or superconducting
generators should be increased efficiency. One of the main
reasons why the efficiency of the superconducting generator
can be increased is because of the elimination of the rotor
excitation loss, which is typically 6 MW for a conventional
1300 MW generator.
The work of the present authors has been confined to
the superconducting generator; the essential components
of which are shown in Fig. 1. A superconducting rotor
winding at 5 K, supported in slots in a stainless-steel rotor
forging, is surrounded by an ambient temperature rotor
screen to protect the superconductor from transient and
negative sequence armature fluxes. The ambient temperature
stator consists of two parts: the first is a self-supporting
structure in which the water-cooled copper armature is
embedded, the second is an iron core, or environmental
screen, which screens the environment from magnetic
fields and increases the radial flux density and conse-
quentially the induced armature e.m.f. per metre length.
This paper is concerned with the properties and design
of the novel helical stator winding in particular.
Helical windings are not new and found limited favour
for dynamos in Germany in the late 1880s and have
since been revived for small d.c. machines of the ironless
type, where they offer the advantage of low inertia and
smooth torque at low speeds. They have also been used for
linear motors and transverse flux tubular motors. The
use in large a.c. machines was first proposed by Ross,
Anderson and Macnab who suggested it as a suitable
armature winding for superconducting a.c. generators.
More recently, a 4 MVA helical armature winding has been
used by Watanabe et al. as the armature winding for use
in either a slotless or a superconducting generator, and
Conley et al. have used a helical winding in an experi-
mental 10 MVA superconducting a.c. generator at MIT.
The helical winding has no distinct division into active
and end regions as has the conventional winding, because
each conductor follows a helical path of constant pitch
from end to end of the machine, see Section 2. In the
conventional winding on the other hand, each conductor
follows an axial path in the active region and only has a
circumferential component to its path in the end regions.
Thus the helical winding is characterised by a circumfer-
ential component of current density, which only decreases
gradually with increasing axial distance from the ends;
instead of being confined to the end winding region as in
the conventional winding. This suggests that there might
be axial flux problems which would not occur with a more
conventional winding. However, work by Tavner et al.
in 1974 has shown that, contrary to what might be expected,
the end-region axial fluxes in the iron core produced by a
helical air-gap winding are less than those produced by a
conventional diamond winding. Such observa-
tions are also supported by the work of Watanabe et al.
The synchronous and subtransient reactances are funda-
mental to generator design and performance and, in order
to design a helical winding, these reactance expressions
must be developed in the simplest possible form. In the
present paper the magnetic field distributions within the
generator which are produced by a helical winding are
obtained in terms of Bessel functions and reactance
expressions are then derived; which account for all the
armature space harmonics. In this way the lowest harmonic
sum that gives satisfactory reactances can be used. An
alternative to the above method is to use a transmis-
sion-line analogue. However, although this method yields
magnetic field values without the introduction of Bessel
functions, the solution is numerical. This is a disadvantage
of the method, since analytic inductance expressions
simplify iterative generator design techniques. More
recently, Alwaisi has developed an analytic 3-dimensional
method which is expressed in Fourier-Bessel expansions.
The solution requires extensive computation for the bound-
ary matchings, although it is capable of achieving high
accuracy.

2 Development of an helical armature winding

As generator sizes increase, the length between bearings
becomes a limitation, see Section 1. Increased output in
larger machines can only be obtained by increasing the
output per unit length by an factor which keeps the
length between bearings down to an acceptable level.
In the case of the conventional machine, electric and magnetic
loadings can only be increased marginally and therefore
the only remaining variable is the effective stator-winding
diameter. Stator-winding diameter is however closely tied
in the rotor diameter because of the need to closely couple
the two windings and so give an acceptable rotor m.m.f.
and stator magnetic loading. Thus since the rotor diameter
is limited by rotational stresses, so too indirectly is the
stator diameter.

On the other hand such restrictions no longer apply to the
superconducting machine, and a considerable decoupling
between rotor and stator windings becomes possible with-
out incurring the same penalties that would arise in the
conventional machine. This is because the superconducting
winding can operate at current densities at least ten times
greater than is current turbogenerator practice and this
in turn, permits magnetic loadings at least as high as those
obtained in the conventional machine, i.e. 0.9T, but at a

substantially increased stator diameter; even though the rotor diameter is still limited by rotational stresses to a value not far different from current practice. Hence the stator diameter can be increased from 1.6 m in the conventional machine to 2.0 m in the superconducting machine, with a consequent increase in the possible output per metre. Furthermore, in the conventional turbogenerator the available winding space is limited by the need to support the windings in slots in the iron core, whereas in the case of the airgap-wound superconducting machine the proportion of the circumference available for the winding increases, as does the possible winding depth; this allows the electric loading to increase from typically 200 kA/m to 300 kA/m in a 1300 MW design. The increased armature-winding diameter and electric loading combine in the case of the superconducting generator to produce a short, large-diameter armature winding with a power output of 300 MW/m instead of the 100 MW/m typical for the conventional turbogenerator.

A secondary result of the absence of clearly defined closed magnetic circuits to constrain the flux is that the distinction between active and end regions becomes blurred.

Because of the way in which the active and end regions merge, it becomes apparent at a fairly early stage in the design that the conventional winding might not necessarily be the best and that alternatives should be examined. If a helical winding is used, then it can be uniformly supported from end to end; a feature which is particularly important now that the armature conductors are no longer shielded from the main flux by the stator teeth and must therefore bear the full machine forces.

To understand how a helical winding works, consider the winding length of a diamond winding to be progressively reduced, as in Fig. 2, by eliminating the so-called central active region and allowing the two diamond-end windings to coalesce and form a helical winding. One phase of a two-pole helical winding is shown in Fig. 3, where each phase consists of two parallel-connected helical loops, each of which is termed a phase group. Each phase group consists of one right-hand (inner) helical phaseband connected in series with a returning left-hand (outer) helical phaseband; a right-hand phaseband being defined as that which traces a right hand screw.

Because the development of large generators in the UK is generally concerned with two-pole 3000 rev/min^3 machines, only two-pole helical windings are considered in the present paper.

3 Method of analysis

3.1 Notation adopted in the description of a helical winding

Each phaseband is described by three parameters:
(a) +1 or -1 depending on whether a right-hand (r.h.) or left-hand (l.h.) helixed phaseband is considered
(b) the circumferential angular position \( \alpha \) at which the centre of the phaseband crosses \( z = 0 \), Fig. 3
(c) the radius of the phaseband \( r \), e.g. \( K_r (r_1, \alpha, +1) \) refers to the axial component of linear current density for the \( i \)th right-hand phaseband centred at angular position \( \alpha \) and radius \( r_1 \).

The angular position of the two phasebands in a phase-group are related as
\[
\alpha_i (-1) = \alpha_i (+1) + \pi
\]

while the angular position of a phasegroup is defined as the angular position of its right-hand phaseband,
\[
\alpha_g = \alpha (+1)
\]

and the angular position of a phase as
\[
\alpha_{ph} = \alpha_g
\]

![Fig. 2](image-url) Evolution of helical winding from diamond lap winding
- a) Diamond lap winding, long machine
- b) Diamond winding, short machine with greater specific machine output
- c) Diamond winding, with straight portion removed to give semi-helical winding
- d) Full helical winding

![Fig. 3](image-url) One phase of helical winding

\[ \alpha_{ph} = \alpha_z - \pi \]  

(3)  

because the complete phase is symmetrical.

3.2 Representation of a helical winding

Each phaseband of the helical armature winding is represented by a linear current density sheet of negligible radial thickness, with axial and tangential linear current-density components as described in Section 11.1. In order to yield a closed analytical solution to Laplace's equation, a periodic variation is introduced in the axial direction in a similar manner to that suggested by Hammond,\(^{23}\) by assuming the helical winding to be one of an infinite number of helical windings placed end to end. Such a winding is termed an 'infinite periodic helical winding'.

3.3 Representation of the environmental screen

A superconducting a.c. generator requires an environmental screen which is normally constructed from laminated magnetic iron, in order to screen the environment from the magnetic fields produced inside the generator. However, conducting environmental screens have been suggested,\(^{19}\) particularly for airborne applications,\(^{22}\) and this analysis contains solutions for both types of screen. The general effect of the iron environmental screen is, as expected, to enhance the main magnetic field, whereas the conducting screen has a demagnetising effect.

The environmental screen is assumed to be infinitely long in the axial direction. If it is constructed from laminated magnetic iron it is assumed to have infinite permeability and zero conductivity, i.e. there are no eddy currents. If on the other hand it is constructed from non-magnetic conducting material, it is assumed to have a relative permeability of unity and infinite conductivity. In either case the effect of the screen is included in the magnetic-field analysis by using an image winding with the appropriate current polarity placed outside the actual helical winding. In the case of the iron environmental screen, the image winding assists the radial field and therefore is of the same polarity as the armature winding. In the case of a conducting screen, the image winding must oppose the radial field and therefore is of opposite polarity to the armature winding.

In practice, the iron environmental screen tends to be shorter than the helical armature winding, as indicated in Fig. 1, and it is likely too that a conducting end screen would be used. Consequently the assumption of an infinitely long environmental screen, whilst necessary to enable a ready solution to the problem, gives a high value of reactance and is mainly responsible for the difference between calculated and measured reactance values. However, in the model results of Section 6, the difference in the synchronous reactance is less than 2%, indicating how little the ends contribute to the reactance.

3.4 Mathematical solution method

The infinite periodic helical winding is analysed in cylindrical polar co-ordinates with the origin of the co-ordinate system taken at the axial centre of the winding such that the actual winding extends from \( z = +1 \) to \( z = -1 \) and constitutes one half-period of the 'infinite periodic helical winding'. The magnetic field due to an individual phaseband at the general circumferential position \( \alpha_z \) is obtained from a solution of Laplace's equation, expressed in terms of magnetic scalar potential, by the method of separation of variables. Magnetic scalar potential terms produced by the image winding which represents the environmental screen are combined with those from the helical armature winding and magnetic-field distributions are then obtained in the manner described in Section 11.2.

4 Magnetic field of a helical winding

The magnetic field distributions for an individual phaseband of an infinite periodic helical winding with an environmental screen are shown in Table 1. The environmental screen modifies the magnetic field distributions of the unscreened winding by the geometric factors defined in Table 2.

The magnetic field distributions described in Table 1 include terms that are independent of axial or tangential position. The effect of these zero-order terms is reflected in the magnetic field distributions as a solenoidal field which is produced by the zero-order component in the tangential linear current density \( K_1(\alpha_z, \alpha_q, \pm 1) \). Because the winding is part of an infinite helix, a magnetic field component is produced only inside the winding, just as though it were an infinite solenoid. The zero-order component of axial linear current density, \( K_0(\alpha_z, \alpha_q, \pm 1) \), produces a tangential magnetic field outside the winding that varies, in accordance with Ampère's law, inversely with radius.

Using superposition to combine a left-hand and a right-hand phaseband to give a phasegroup, as in Table 3, eliminates the zero-order tangential magnetic field. This is because the zero-order component of axial current density in the right-hand phaseband, \( K_1(\alpha_z, \alpha_q, +1) \), opposes the equivalent component in the left-hand phaseband \( K_1(\alpha_z, \alpha_q, -1) \), as shown in Fig. 4. However, because the corresponding zero-order components of tangential linear current density, \( K_0(\alpha_z, \alpha_q, +1) \) and \( K_0(\alpha_z, \alpha_q, -1) \) are both in the same direction their effect is additive and the solenoidal magnetic field is increased. This is also shown schematically in Fig. 4. When two phasegroups are combined to produce a phase, the relative phaseshift of 180° which exists between the two phasegroups ensures that this solenoidal magnetic field is eliminated. The influence of this effect on measured and computed inductions is described in Section 6.

5 Reactance calculations

5.1 Synchronous reactance

Reactance expressions are obtained by assuming that the armature winding is concentrated at the geometric mean winding radius \( r_s \) and by only taking account of the fundamental time harmonic.

Consider the two helical loops shown in Fig. 5; one centred at \( \theta = 0 \) and the other at \( \theta = \alpha_z \) and assume there to be a phasegroup of \( Z/b \) turns centred about the second loop. The flux from this phasegroup which links the first helical turn is, by Gauss's theorem, equal to the flux that crosses normally any surface bounded by the first, like that shown in Fig. 5. Evaluating the resulting surface integrals as described in Section 11.4 allows the inductance between phasegroups to be expressed as in eqn. 51. If the phasegroup inductance values are combined appropriately, phase and generator inductances are obtained, see eqns. 53 and

\[ \text{IEE PROC., Vol. 127, Pt. C, No. 3, MAY 1980} \]
Table 1: Magnetic field distributions produced by a helical phase-bound

\[ K_a[r, a, \phi, z] = \frac{Z_\text{rho}_a}{2ab} + Z_\text{rho}_a \sum_{n=1}^{\infty} k_{bn} \cos n(\theta - \phi + k z) \]

\[ K_b[r, a, \phi, z] = \frac{Z_\text{rho}_b}{2ab} + Z_\text{rho}_b \sum_{n=1}^{\infty} k_{bn} \cos n(\theta - \phi + k z) \]

Inside the winding \( r < r_i \)

\[ \Omega[r, a, \phi, z] = \frac{Z_\text{rho}_a}{2ab} + Z_\text{rho}_a \sum_{n=1}^{\infty} k_{bn} \cos n(\theta - \phi + k z) \]

\[ H_z[r, a, \phi, z] = \frac{Z_\text{rho}_a}{2ab} + Z_\text{rho}_a \sum_{n=1}^{\infty} k_{bn} \cos n(\theta - \phi + k z) \]

\[ H_y[r, a, \phi, z] = \frac{Z_\text{rho}_a}{2ab} + Z_\text{rho}_a \sum_{n=1}^{\infty} k_{bn} \cos n(\theta - \phi + k z) \]

Outside the winding \( r_i < r < r_x \)

\[ \Omega[r, a, \phi, z] = \frac{Z_\text{rho}_a}{2ab} + Z_\text{rho}_a \sum_{n=1}^{\infty} k_{bn} \cos n(\theta - \phi + k z) \]

\[ H_z[r, a, \phi, z] = \frac{Z_\text{rho}_a}{2ab} + Z_\text{rho}_a \sum_{n=1}^{\infty} k_{bn} \cos n(\theta - \phi + k z) \]

\[ H_y[r, a, \phi, z] = \frac{Z_\text{rho}_a}{2ab} + Z_\text{rho}_a \sum_{n=1}^{\infty} k_{bn} \cos n(\theta - \phi + k z) \]

Environmental screen factors \( k_{mn}, k_{pn}, k_{sn} \) defined in Table 2

\[ \delta_n(nkr) = [K_n(nkr) - \gamma_n(nkr)] \]

55. The synchronous reactance of a helical winding, given in eqn. 55, can be expressed in the more usual form as

\[ x_m = \frac{m}{n} \omega L_s T_s^2 k_{s}^2 k_{m}^2 \text{ ohms} \]

\[ n = 2 \pi m \pm 1, \tau \text{ any integer} \]

with the winding factor

\[ k_{s}^2 \text{ is the skew factor defined by eqn. 48 and } K_n \text{ the } n \text{th harmonic radial flux environmental screen enhancement factor defined in the first column of Table 2.} \]

5.2 Subtransient and transient reactance

Under fault conditions the armature flux is initially deflected around the outside of the outer rotor, as if this outer rotor were a perfectly conducting screen. This effect is included in the analysis by representing the outer rotor screen by an image winding at the surface of the outer rotor, which is equal in magnitude, but opposite in phase, to the armature current. This current introduces an additional magnetic scalar potential term into the magnetic field solution as described in Section 1.4.4, such that the resulting integration over the Gaussian surface of Fig. 5 gives for the subtransient reactance:

\[ x_{s}^{\prime} = \frac{m}{n} \omega L_s T_s^2 k_{s}^2 k_{m}^2 \text{ ohms} \]

\[ 1 - \frac{\gamma_n(nkr)}{\gamma_m(nkr)} \]

where the factors \( \gamma_n(nkr) \) and \( \beta_n(nkr) \) are defined in eqns. 43' and 44b, respectively. This expression is similar to that for the synchronous reactance except for the geometric modifying factors, both of which depend, through Bessel functions, on the armature length. The two modifying factors relate to the different generator components as

\[ \begin{bmatrix} 1 - \frac{\gamma_n(nkr)}{\gamma_m(nkr)} \\ \frac{\beta_n(nkr)}{\gamma_m(nkr)} \end{bmatrix} \]
a geometric factor relating armature and outer rotor

\[
\left[ 1 - \frac{\beta_{h}(nk_{s})}{\gamma_{h}(nk_{s})} \right]
\]

a geometric factor relating the iron environmental screen and the outer rotor.

(Note: For a conducting environmental screen \( \gamma_{h}(nk_{s}) \) replaces \( \beta_{h}(nk_{s}) \). Table 4 details all the permutations.)

As the fault period progresses flux will penetrate the outer rotor and two further reactance expressions require definition; the first being associated with the radiation screen, when flux is deflected round the outside of this screen, and the second, the transient reactance, when flux is deflected round the outside of the rotor field winding. These can be obtained from eqn. 6 by substituting the outer rotor radius \( r_{o} \), by the radiation screen radius \( r_{e} \), and the superconducting field winding radius \( r_{f} \), respectively. The transient reactance is unusual in the superconducting generator because it is associated with a transient open-circuit time constant greater than 100's compared with one of less than 10's in the conventional turbogenerator.

5.3 Effect of harmonics on reactance calculation

The reactance between phases and the synchronous reactance are computed for the 1300 MW design of Table 5. This generator has a six-phase armature winding with opposite phasegroups connected in parallel. The results of the reactance calculations are shown in Table 6 first for the fundamental harmonic only and secondly for the harmonic sum of \( n = 1 \) to 50.

Table 2: Expressions for \( n \)th space harmonic environmental screen factor

<table>
<thead>
<tr>
<th>Inside stator ( r &lt; r_{e} )</th>
<th>Outside stator ( r_{e} &lt; r &lt; r_{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{s_{m}} ) ( 1 + \frac{\beta_{m}(nk_{s})}{\gamma_{m}(nk_{s})} ) ( 1 - \frac{\beta_{m}(nk_{s})}{\gamma_{m}(nk_{s})} )</td>
<td></td>
</tr>
<tr>
<td>( k_{s_{p}} ) ( 1 + \frac{\beta_{p}(nk_{s})}{\gamma_{p}(nk_{s})} ) ( 1 - \frac{\beta_{p}(nk_{s})}{\gamma_{p}(nk_{s})} )</td>
<td></td>
</tr>
<tr>
<td>( k_{s_{n}} ) ( 1 + \frac{\beta_{n}(nk_{s})}{\gamma_{n}(nk_{s})} ) ( 1 - \frac{\beta_{n}(nk_{s})}{\gamma_{n}(nk_{s})} )</td>
<td></td>
</tr>
</tbody>
</table>

Iron environmental \( k_{s_{m}} \), Conducting environmental \( k_{s_{p}} \), and \( k_{s_{n}} \) are all shown for each phase in Tables 6.

If no environmental screen \( k_{s_{m}} = k_{s_{p}} = k_{s_{n}} = 1 \)

5.4 Comparison of reactance of a helical winding with those of a straight winding

By assuming the armature winding to be infinitely long and each phase to be represented by a current sheet of the form

\[
\frac{2T_{p_{b}}k_{s_{m}}}{\pi r_{e}} \sin \frac{n \pi d}{\lambda_{p}} \text{ A/m}
\]

the synchronous reactance is

\[
x_{s_{m}} = \frac{m}{r_{e}} \omega_{o} T_{p_{b}} k_{s_{m}} k_{r_{m}} \text{ 3} \text{ ohms}
\]

where \( k_{r_{m}} \), the winding factor is

\[
k_{r_{m}} = k_{s_{m}} k_{p_{m}}
\]

Fig. 4 Zero order components of phaseband and phasegroup linear current density distributions

a Zero order components of right-hand phaseband
b Zero order components of left-hand phaseband
c Zero order components of phasegroup

Table 3: Magnetic field distributions produced by a helical phasor group

Linear current density distributions

\[ K_{q1}(r_1, \alpha_1) = \frac{2Z_{phq} \pi}{s nab} \sum_{n \text{ odd}}^\infty k_{bn} \cos n(\theta - \alpha_1) \cos nkr \cos nkrx + \frac{2Z_{phq} \pi}{s nab} \sum_{n \text{ even}}^\infty k_{bn} \sin n(\theta - \alpha_1) \sin nkr \sin nkrx \]

\[ K_{q2}(r_1, \alpha_1) = \frac{Z_{phq}}{2ab} + \frac{Z_{phq}}{lab} \sum_{n \text{ odd}} k_{bn} \sin n(\theta - \alpha_1) \sin nkr \cos nkrx + \frac{Z_{phq}}{lab} \sum_{n \text{ even}} k_{bn} \cos n(\theta - \alpha_1) \cos nkr \cos nkrx \]

Inside the winding \( r < r_1 \)

\[ \Omega_1(r_1, \alpha_1) = -\frac{Z_{phq} r}{2ab} + \frac{2Z_{phq}}{s nab} \sum_{n \text{ odd}}^\infty k_{bn} \frac{k_{bn} \sin nkr_1 + k_{bn} \sin nkr_1}{\delta_n(nkr_1)} \cos nkr \cos nkrx + \frac{2Z_{phq}}{s nab} \sum_{n \text{ even}}^\infty k_{bn} \frac{k_{bn} \sin nkr_1 - k_{bn} \sin nkr_1}{\delta_n(nkr_1)} \sin nkr \sin nkrx \]

\[ H_{q1}(r_1, \alpha_1) = \frac{2Z_{phq}}{s nab} \sum_{n \text{ odd}}^\infty k_{bn} K_1(nkr_1) \left[ -k_{bn} \sin nkr_1 + k_{bn} \cos nkr_1 \right] \cos nkr \cos nkrx + \frac{2Z_{phq}}{s nab} \sum_{n \text{ even}}^\infty k_{bn} K_1(nkr_1) \left[ -k_{bn} \cos nkr_1 - k_{bn} \sin nkr_1 \right] \sin nkr \sin nkrx \]

\[ H_{q2}(r_1, \alpha_1) = \frac{2Z_{phq}}{s nab} \sum_{n \text{ odd}}^\infty k_{bn} K_1(nkr_1) \left[ -k_{bn} \sin nkr_1 + k_{bn} \cos nkr_1 \right] \cos nkr \cos nkrx + \frac{2Z_{phq}}{s nab} \sum_{n \text{ even}}^\infty k_{bn} K_1(nkr_1) \left[ -k_{bn} \cos nkr_1 - k_{bn} \sin nkr_1 \right] \sin nkr \sin nkrx \]

Outside the winding \( r_1 < r < r_2 \)

\[ \Omega_2(r_2, \alpha_1) = +\frac{2Z_{phq}}{s nab} \sum_{n \text{ odd}}^\infty k_{bn} \frac{k_{bn} K_{-1}(nkr_1)}{\delta_n(nkr_1)} \cos nkr \cos nkrx + \frac{2Z_{phq}}{s nab} \sum_{n \text{ even}}^\infty k_{bn} \frac{k_{bn} K_{-1}(nkr_1)}{\delta_n(nkr_1)} \sin nkr \sin nkrx \]

\[ H_{q1}(r_2, \alpha_1) = -\frac{2Z_{phq}}{s nab} \sum_{n \text{ odd}}^\infty k_{bn} K_{-1}(nkr_1) \left[ k_{bn} \cos nkr_1 + k_{bn} \cos nkr_1 \right] \cos nkr \cos nkrx + \frac{2Z_{phq}}{s nab} \sum_{n \text{ even}}^\infty k_{bn} K_{-1}(nkr_1) \left[ k_{bn} \sin nkr_1 - k_{bn} \sin nkr_1 \right] \sin nkr \sin nkrx \]

\[ H_{q2}(r_2, \alpha_1) = +\frac{2Z_{phq}}{s nab} \sum_{n \text{ odd}}^\infty k_{bn} K_{-1}(nkr_1) \left[ k_{bn} \cos nkr_1 + k_{bn} \cos nkr_1 \right] \cos nkr \cos nkrx + \frac{2Z_{phq}}{s nab} \sum_{n \text{ even}}^\infty k_{bn} K_{-1}(nkr_1) \left[ k_{bn} \sin nkr_1 - k_{bn} \sin nkr_1 \right] \sin nkr \sin nkrx \]

\[ H_{q2}(r_2, \alpha_1) = -\frac{Z_{phq}}{lab} \sum_{n \text{ odd}} k_{bn} K_{-1}(nkr_1) \left[ k_{bn} \cos nkr_1 + k_{bn} \cos nkr_1 \right] \cos nkr \cos nkrx + \frac{Z_{phq}}{lab} \sum_{n \text{ even}} k_{bn} K_{-1}(nkr_1) \left[ k_{bn} \sin nkr_1 - k_{bn} \sin nkr_1 \right] \sin nkr \sin nkrx \]

Environmental screen factors \( k_{bn}, k_{bn}, K_{bn}, K_{bn} \) defined in Table 2

\[ \delta_n(nkr) = [K_n(nkr_1) - \gamma_n(nkr_1) l_n(nkr_1)] \]

Table 4: Expressions for the nth space harmonic combined environmental and rotor screen factors

<table>
<thead>
<tr>
<th>Inside stator ( r_2 &lt; r &lt; r_3 )</th>
<th>Outside stator ( r_3 &lt; r &lt; r_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_m )</td>
<td>( \frac{1 + \frac{a}{\gamma_m(nkr)}}{1 - \frac{a}{\gamma_m(nkr)}} )</td>
</tr>
<tr>
<td>( \frac{1 - \frac{a}{\gamma_m(nkr)}}{1 + \frac{a}{\gamma_m(nkr)}} )</td>
<td></td>
</tr>
<tr>
<td>( k_{bn}, K_{bn} )</td>
<td>( \frac{1 - \frac{a}{\beta_m(nkr)}}{1 + \frac{a}{\beta_m(nkr)}} )</td>
</tr>
<tr>
<td>( \frac{1 - \frac{a}{\beta_m(nkr)}}{1 + \frac{a}{\beta_m(nkr)}} )</td>
<td></td>
</tr>
</tbody>
</table>

\( a = \beta_m(nkr_2) \) for iron environmental screen
\( a = \gamma_m(nkr_2) \) for conducting environmental screen
\( a = 0 \) for no environmental screen
\( b = \beta_m(nkr_2) \) for iron rotor screen
\( b = \gamma_m(nkr_2) \) for conducting rotor screen
\( b = 0 \) for no rotor screen


135
\[ k_{m} \] being the \( n \)th harmonic pitch factor. This equation is of the same form as that for a helical winding, eqn. 4, except for the winding factor, \( k_{w} \), and environmental screen factor \( k_{s} \). The environmental screen factors obtained by the two dimensional analysis outlined above, see References 23, 24 and 6, are shown in Table 2 along with the corresponding values for the helical winding, the fundamental difference between the environmental screen factors being the dependence on winding length introduced by the helical winding. However, finite-length effects become less prominent as the length/diameter ratio of the helical winding increases. Hence, as this ratio tends to infinity (i.e. as the winding length tends to infinity) the environmental screen factors for the helical winding tend to become the same as those for a long straight winding. In the same way, the modifying factors associated with the subtransient reactance of the helical winding, defined in eqn. 6, tend to

\[
\left[ 1 \pm \left( \frac{r_{a}}{r_{b}} \right)^{2n} \right] \left[ 1 \pm \left( \frac{r_{a}}{r_{c}} \right)^{2n} \right]
\]

(+) iron environmental screen
(−) conducting environmental screen

as the winding length tends to infinity. This is identical with the expression obtained for the long straight winding with a conducting environmental screen by Appleton and Anderson and an iron environmental screen by Miller and Hughes.25

Unlike the environmental screen factors, which tend to the same value as winding length increases, the skew factor, \( k_{s} \), of the helical winding does not tend to 1 as the armature length tends to infinity, but to 0.5. This arises because, whatever the length of the generator, a proportion of the flux must be lost due to skewing of the conductors through 180° over the total generator length. The variation of skew factor \( k_{s} \) with length/diameter ratio is shown in Fig. 6.

One effect of skewing the armature conductors is to reduce the amount of rotor flux linking the armature winding, and consequently the induced e.m.f. However, as the rotor flux density at the armature winding falls off towards the ends of the machine, the majority of the generated e.m.f. is towards the centre of the machine. Hence in practical designs this loss of flux is found to be less than 10%.

This loss of flux in the helical winding is compensated for by the superior winding support and by the reduction of synchronous reactance (and in consequence the reduced armature reaction). This drop in synchronous reactance arises because the skew factor for a helical winding is less than unity and because the environmental screen factor \( k_{m} \) is also less than 1.

\( k_{s} \) would be for a straight winding, \( k_{m} \) has a value of 1.54 for a straight winding and 1.38 for the helical winding design of Table 5.

### 6 Comparison of inductance calculations with model results

To assess the accuracy of the inductance expressions of

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVA rating</td>
<td>1330 MVA</td>
</tr>
<tr>
<td>Power factor</td>
<td>0.85 lagging</td>
</tr>
<tr>
<td>Terminal voltage (r.m.s./phase)</td>
<td>16 kV</td>
</tr>
<tr>
<td>Armature current (r.m.s./phase)</td>
<td>15.04 kA</td>
</tr>
<tr>
<td>No. phases</td>
<td>6</td>
</tr>
<tr>
<td>No. parallel paths/phase</td>
<td>2</td>
</tr>
<tr>
<td>No. phases/phasegroup</td>
<td>2</td>
</tr>
<tr>
<td>No. phasegroups/phase</td>
<td>2</td>
</tr>
<tr>
<td>No. conductors/phase</td>
<td>48</td>
</tr>
<tr>
<td>Field-winding radius</td>
<td>0.423 m</td>
</tr>
<tr>
<td>Radiation-screen radius</td>
<td>0.85 m</td>
</tr>
<tr>
<td>Outer-rotor inner radius</td>
<td>0.515 m</td>
</tr>
<tr>
<td>Outer-rotor outer radius</td>
<td>0.65 m</td>
</tr>
<tr>
<td>Armature mean winding radius</td>
<td>1.03 m</td>
</tr>
<tr>
<td>Inner radius of iron environmental screen</td>
<td>1.4 m</td>
</tr>
<tr>
<td>Armature half-length</td>
<td>2.9 m</td>
</tr>
</tbody>
</table>

#### Table 6: Calculated reactance values for helical winding with iron environmental screen, showing effect of space harmonics

| Harmonic | Reactance \( wL_{ph} \) \( wM_{ph}(n/6) \) \( wM_{ph}(n/3) \) \( x_{p} \) |
|----------|-----------------|-----------------|-----------------|-----------------|
| Sum \( n = 1 \) to 50 | 0.224 | 0.128 | 0.056 | 0.5018 |
| Sum \( n = 1 \) | 0.167 | 0.145 | 0.084 | 0.5013 |

#### Table 7: Effect of harmonics on the breadth factor of 6-phase and 3-phase windings

<table>
<thead>
<tr>
<th>6-phase</th>
<th>3-phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( k_{b}^{6p} )</td>
</tr>
<tr>
<td>1</td>
<td>0.97726</td>
</tr>
<tr>
<td>11</td>
<td>0.00723</td>
</tr>
<tr>
<td>13</td>
<td>0.00844</td>
</tr>
<tr>
<td>23</td>
<td>0.00900</td>
</tr>
</tbody>
</table>

Section 5.1, extensive measurements have been made using a 1:9 scale model of a 1300 MW turbogenerator at Leeds University. The helical winding model has a mean radius of 105 mm and length of 406 mm and is shown in Fig. 7. To simplify construction, it employs comparatively few conductors, 24 per layer, which are disposed in three phases with opposite phasegroups connected in series to minimise errors due to circulating currents. The inductance between individual turns can be measured. The iron environmental screen model, shown in Fig. 6, is manufactured from laminated iron and has an inner radius of 134.5 mm and length of 356 mm. The ends of the environmental screen are manufactured from aluminium with an inner radius of 150 mm and extended 88 mm to the aluminium end plates of the model. The relevant dimensions of the model are summarised in Table 8.

Measured and computed values of interturn mutual inductance, both for the helical winding in air and inside the iron environmental screen, are shown as a function of coil separation in Figs. 9 and 10, respectively. Computed inductance values are obtained by assuming a phase spread of 15° (the total spread of each conductor) and summing to a harmonic order of 81; this being the maximum order accommodated by the computation.

Fig. 6 Graph of skew factor $k_{sk}$ against $k_{r}$ for helical winding

Fig. 7 Model helical winding

Fig. 8 Iron environmental screen

Fig. 9 Interturn mutual inductance as function of separation for model winding in air

Fig. 10 Interturn mutual inductance as a function of separation for model helical winding inside iron environmental screen

Table 8: Model helical winding dimensions

<table>
<thead>
<tr>
<th>Helical winding:</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st-layer radius</td>
<td>100</td>
</tr>
<tr>
<td>2nd-layer radius</td>
<td>110</td>
</tr>
<tr>
<td>Winding length</td>
<td>150</td>
</tr>
<tr>
<td>Conductor diameter</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Iron Core:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner radius</td>
<td>134.5</td>
</tr>
<tr>
<td>Length</td>
<td>360</td>
</tr>
</tbody>
</table>

The presence of the 'solenoidal' inductance term in the phase group inductances and interturn inductances, makes comparison of measured and computed phase group, or interturn, inductance values difficult because the solenoidal inductance value computed by the 'infinite periodic helical winding' is that of an infinite solenoid, whereas that measured is of a solenoid of finite length. The latter can be substantially different from the former. The interturn mutual inductances measured in air (Fig. 9), however, show very close agreement with the calculated values, being on average higher by 10% of the solenoidal inductance term. These results suggest that the inevitable overestimation of solenoidal inductance has been countered by main flux linkage around the winding ends. When measured in the laminated iron core (Fig. 10) a more pronounced discrepancy, on average 50% of the solenoidal inductance, is shown. This arises because the model core is shorter than the winding and the boundary extended to the machine end plates by aluminium rings of inner radius 150 mm. Consequently, any measured results may be expected to be less than those predicted by the infinite periodic model with the infinitely long environmental screen. Also shown in Fig. 10 is an interturn inductance computation using the fundamental harmonic component only. This clearly demonstrates the high space harmonic content of interturn, or equally phase group, mutual inductances.

A comparison of self-inductances is shown in Table 9 and gives two measured and two calculated values of inductance for the winding in air and inside the laminated iron core. The first column gives directly measured values, whilst the second gives values derived from measurements on the previous subgroup. For example, the phase self-inductance was derived from the measurements on 60\(^\circ\) spread phase groups, showing a discrepancy of 2-5% with the directly measured value. This discrepancy, absent from mutual values, is attributable to the conductor necessary to form the series connection spanning 180\(^\circ\). The prominence of this effect is because of the air-cored nature of the geometry, even in the iron core, and exaggerated in the model by the small number of conductors, 16, that form a phase. The computed values of self-inductance depend on conductor size. The first column in Table 9 assumes a full spread for the current sheet of 15\(^\circ\) per turn, whereas in the second column a spread of 7.5\(^\circ\) has been used. The true angle subtended by the conductor is approximately 3\(^\circ\), but if this spread is used with infinitesimal depth then excessively high inductance values result. In other words if a continuous current sheet is used to represent discrete conductors, a fair model results.

The error between computed and measured values in Table 9 is in all cases less than 10% and the synchronous reactance values agree to within 2%. The final row of Table 8 uses only the fundamental component in eqn. 4 to compute synchronous reactance and gives results to within 2%, indicating that only the fundamental component need be considered when computing synchronous and subtransient reactances.

7 Design of a helical armature winding for use in a superconducting a.c. generator

For the generator design of Table 5, the radial flux density produced by the superconducting field winding at the armature in the presence of an iron environmental screen was computed by GFUN\(^2\) with account being taken of environmental screen length, and the length, depth and slotting arrangement of the superconducting field winding. The computed radial flux density variations are shown as a function of angular position and axial position in Figs. 11 and 12, respectively.

Analysis of the variations in radial flux density with tangential position at the armature radius indicates a maximum harmonic content at the axial centre of the machine of under 1%. Hence the e.m.f. induced in a phase can be assumed to be due to the fundamental harmonic only and is given by

\[
E = \frac{\mu_0 c a^2 \psi^2}{\sqrt{2a}} \int_0^a B_x(x) \cos \frac{\pi x}{a} \, dx \,\text{volts} \tag{10}
\]

where the variation of radial flux density with axial position \(B_x(x)\) is shown in Fig. 12.

Computation of the e.m.f. requires numerical integration of the axial variation of radial flux density produced by the field winding, \(B_x(x)\), while the length and diameter of the helical winding determines the machine inductances.
through eqns. 4 and 6. For a correct design, at a presellected terminal voltage and current, the two quantities of induced voltage and synchronous reactance will satisfy the general phasor diagram shown in Fig. 13.

To design an armature winding, the generator dimensions, reactances and electric and magnetic loadings are obtained using an iterative design technique which assumes the generator windings to be infinitely long and to be represented by fundamental harmonic current sheets at their respective geometric mean winding radii; this reduces the magnetic field problem to two dimensions. A field-winding length is then selected and a flux plot such as that shown in Fig. 12 obtained for the three-dimensional generator configuration. An iterative computer program then computes the induced e.m.f. using numerical integration of eqn. 10 and an estimate of the induced e.m.f. \( E^* \) is obtained from the phasor diagram as

\[
E^* = \sqrt{1 + 2x_a \sin \phi + x_a^2} \text{ p.u.}
\]  

(11)

For a correct design the e.m.f. \( E \) and its estimate \( E^* \) must be equal. If \( E \) and \( E^* \) are not comparable, then armature length and/or the number of armature conductors are varied until a solution is obtained and the subtransient reactance obtained by eqn. 6 is equal to, or greater than, the value computed in the two-dimensional design. This ensures that outer rotor stresses during a short circuit are acceptable. If varying both armature length and the number of conductors will not yield a satisfactory solution, then the length of the rotor field winding must be changed and the iteration repeated.

For an armature of given length the helical winding has been shown to have a lower synchronous reactance (higher short-circuit ratio) than that predicted by two-dimensional analysis for a section of an infinite straight winding of the same length. Hence armature reaction effects are reduced with a helical winding.

8 Conclusion

A closed analytical solution has been presented for the magnetic field distribution and machine reactances of a superconducting a.c. machine with a helical armature winding. There is a significant harmonic content in the phase group and the phase inductances, but when all three phases are combined to give the synchronous, subtransient and transient inductances of the harmonics cancel, except those of order \( n = 2m \pm 1 \), where \( m \) is the number of phases and \( \tau \) any positive integer. Hence the important synchronous, subtransient and transient inductances are largely dominated by the fundamental and therefore can be readily calculated. The reactances can be expressed in a similar form to those for a section of an infinite straight winding, but are modified by two factors, namely a skew factor \( k_{1m} \) and an environmental screen factor \( k_{em} \), both of which contain Bessel functions that depend on the ratio of length to diameter \( (k_{1m} = 1 \text{ for a straight winding}) \). Laboratory measurements on a model helical winding indicate that the synchronous reactance expressions are accurate to within 2%.

9 Acknowledgments

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B.I. Hassall is much indebted to the Science Research Council for financial support.
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11 Appendices

11.1 Linear current density distribution

11.1.1 Notation: The total linear current-density distribution of a phaseband in terms of its axial and tangential linear current density components is

\[ K_1 [r_i, \alpha_i, \pm \pi] = K_0 [r_i, \alpha_i, \pm \pi] a_0 + K_2 [r_i, \alpha_i, \pm \pi] a_2 \]

while the linear current density distribution of a phaseband, with phasebands on different radii is

\[ K_2 [r_i, \alpha_i] = K_1 [r_i, \alpha_i] + K_2 [r_i, \alpha_i + \pi] \]

For a phaseband centred at \( \alpha_i + \pi \), the current flow is normally reversed such that the linear current density distribution of a phaseband is

\[ K_{ph2} [r_i, \alpha_i] = K_2 [r_i, \alpha_i] - K_2 [r_i, \alpha_i + \pi] \]

The phase current is given by

\[ I_{ph} = \frac{I_{ph}}{b} \sin \left( \phi - \phi - \phi \right) \]

and the total current per phaseband is

\[ I_0 = \frac{Z_{ph}}{b} \]

A phaseband current distribution of magnitude \( K \), spread \( \alpha \), and centred at \( \phi \), can be represented by a periodic pulsed square wave with Fourier series

\[ f(\phi) = \frac{K_0}{2a} + \sum_{n=1}^{\infty} \frac{2K}{n\pi} \sin \left( \frac{n\phi}{2} \right) \cos n(\theta - \phi) \]

Fig. 3 shows that a right-hand phaseband advances linearly with \( \theta \) such that

\[ \phi = k_1 + \alpha \]

Likewise for a left-hand phaseband

\[ \phi = -k_1 + \alpha \]

Since the total machine length is \( 2\pi \) over which a phaseband circumscribes one pole pitch the helix periodicity is \( k = \pi/2 \).

11.1.2 Axial linear current-density distribution: If the phaseband subtends a phaseband \( \alpha \) the circumferential distance occupied by the phaseband is \( \alpha r_0 \) and

\[ K_2 [r_i, \alpha_i, \pm \pi] = \frac{1}{\alpha r_0} \frac{Z_{ph}}{2ab} \sum_{n=1}^{\infty} k_n \cos \left( \phi - \phi + \phi \right) \]

Substituting in eqn. 17 and for \( f(\theta) \)

\[ K_2 [r_i, \alpha_i, \pm \pi] = \frac{Z_{ph}}{2ab} \sum_{n=1}^{\infty} k_n \cos \left( \theta - \phi + \phi \right) \]

The linear current density for the corresponding \( n \)th phaseband is from eqn. 15, with \( r_i = r_f \)
\[ K_{\text{ph}}[r_i, \alpha_i] = \frac{2Z_{\text{ph}}}{ab\tau_r} \sum_{n=1}^{\text{n odd}} k_{bn} \cos n(\theta - \alpha_i) \cos nkz \]
with the corresponding phase value
\[ K_{\text{ph}}[r_i, \alpha_i] = \frac{4Z_{\text{ph}}}{ab\tau_r} \sum_{n=1}^{\text{n odd}} k_{bn} \cos n(\theta - \alpha_i) \cos nkz \]  
(22)

11.1.3 Tangential linear current density distribution: Tangential linear current density is again a pulse wave advancing in the z-direction for increasing \( \theta \), i.e., \( \phi_l = \pm kx + \alpha_i \), with wave period \( 4l \) implying
\[ K_{\theta}[r_i, \alpha_i, \pm 1] = \frac{Z_{\text{ph}}}{2ab\tau} \]  
giving after substitution in eqn. 17 for the phaseband
\[ K_{\theta}[r_i, \alpha_i, \pm 1] = \frac{Z_{\text{ph}}}{4ab} \]
(24)
and for a phasegroup
\[ K_{\theta}[r_i, \alpha_i] = \frac{Z_{\text{ph}}}{2ab} \sum_{n=1}^{\text{n odd}} k_{bn} \cos n(\theta - \alpha_i + kxz) \]
(25)
and finally for a phase
\[ K_{\text{ph}}[\alpha_i, r_i] = \frac{2Z_{\text{ph}}}{ab\tau} \sum_{n=1}^{\text{n odd}} k_{bn} \sin n(\theta - \alpha_i) \sin nkz \]  
(26)

11.2 Solution of Laplace's equation
11.2.1 Winding in air: Laplace's equation in cylindrical co-ordinates is
\[ \frac{\partial^2 \Omega}{\partial r^2} + \frac{1}{r} \frac{\partial \Omega}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Omega}{\partial \phi^2} + \frac{\partial^2 \Omega}{\partial z^2} = 0 \]  
(28)
A solution of the form
\[ \Omega = F_1(\phi)F_2(z) \]  
(29)
can be obtained by the separation of variables such that
\[ \Omega_{\text{ph}} = (A_{\alpha} + A_{\alpha})(A_{\alpha} + A_{\alpha})(A_{\alpha} + A_{\alpha} \ln(r)) \]
(30a)
\[ \mu = v = 0 \]
(30b)
Introducing the boundary conditions of Section 11.3 gives
(a) inside the winding
\[ \Omega_n = A_0 I_n(\kappa r) \cos n(\theta - kx) + \psi_{2n} \]  
(31a)
\[ \Omega_n = A_0 I_n(\kappa r) \cos n(\theta + kx) + \psi_{1n} \]  
(31b)
(b) outside the winding
\[ \Omega_n = B_n K_n(\kappa r) \cos n(\theta - kx) + \psi_{2n} \]  
(32a)
\[ \Omega_n = B_n K_n(\kappa r) \cos n(\theta + kx) + \psi_{1n} \]  
(32b)
\[ \Omega_n = A_0 \theta \]  
(33b)
before the introduction of the environmental screen.

11.2.2 Modification of the potential distribution by the environmental screen: The image winding produces a potential distribution
\[ \Omega_n = D_n I_n(\kappa r) \]
(33)
which is incorporated in the potential distributions (eqns. 31 and 32) as,
(a) inside the winding
\[ \Omega_n = (A_{\alpha} + A_{\alpha}) I_n(\kappa r) \cos n(\theta - kx) + \psi_{2n} \]  
(34a)
\[ \Omega_0 = A_0 \theta \]  
(34b)
(b) outside the winding
\[ \Omega_n = B_n K_n(\kappa r) + D_n I_n(\kappa r) \cos n(\theta - kx) + \psi_{2n} \]  
(35a)
\[ \Omega_0 = A_0 \theta \]  
(35b)
\[ D_n \text{ is determined by the boundary condition (iv), Section 11.3, and describes the type of environmental screen used.} \]

11.2.3 Magnetic field distributions: The separate components are obtained from \( H = \nabla \Omega \) to give:
(i) Radial fields
\[ \mu \ll r \ll \mu \]
\[ H_{rn} = -[A_{\alpha} + A_{\alpha}] I_n(\kappa r) \cos n(\theta - kx) + \psi_{2n} \]  
(36)
\[ H_{rn} = -[B_n K_n(\kappa r) + D_n I_n(\kappa r)] \cos n(\theta + kx) + \psi_{1n} \]  
(37)
\[ r_i \gg r \gg r_s \]
\[ H_{rn} = -B_n K_n(\kappa r) + D_n I_n(\kappa r) \cos n(\theta + kx) + \psi_{1n} \]  
(38)
(ii) Tangential fields
\[ r \ll r_s \]
\[ H_{\text{th}} = (A_{\alpha} + A_{\alpha}) I_n(\kappa r) \cos n(\theta - kx) + \psi_{2n} \]  
(39)
\[ r \ll r_s \]
\[ H_{\text{th}} = (A_{\alpha} + A_{\alpha}) I_n(\kappa r) \cos n(\theta - kx) + \psi_{2n} \]  
(40)
\[ r_0 > r > r_e \]
\[ H_{0n} = \frac{[B_{0n}K_n(nkr) + D_{0n}I_n(nkr)]}{r} \left[ \sin \left( n(\theta - kz) + \psi_{2n} \right) - \sin \left( n(\theta + k\bar{z}) + \psi_{1n} \right) \right] \]
\[ H_{0\theta} = -\frac{A_{0n}}{r} \frac{A_{0n}}{n} \]

(iii) Axial fields
\[ r < r_e \]
\[ H_{0n} = -[A_0 + D_nI_n(nkr)\frac{k}{n}] \left[ \sin \left( n(\theta - k\bar{z}) + \psi_{2n} \right) - \sin \left( n(\theta + k\bar{z}) + \psi_{1n} \right) \right] \]
\[ H_{0\theta} = -\frac{A_{0n}}{\nu} \]

where
\[ I_n^{(nkr)} = \frac{d}{dr} I_n(nkr) = nk \left[ I_{n-1}(nkr) - \frac{nI_n(nkr)}{nkr} \right] \]
\[ K_n^{(nkr)} = \frac{d}{dr} K_n(nkr) = -nk \left[ K_{n-1}(nkr) + \frac{nK_n(nkr)}{nkr} \right] \]

Substitution of the boundary conditions of Section 11.3 gives
\[ A_n = \gamma_n(nkr)B_{0n} \]
\[ \psi_{2n} = \psi_{1n} = -n\alpha + \pi/2 \]
\[ A_{0n} = \frac{\pi I_n^2}{2\pi ab} \]
\[ A_{0\theta} = -\frac{\pi I_n^2}{4\pi ab} \]
\[ B_n[r_t, \alpha_t, \pm 1] = \frac{\pm zK_{bn}(nkr)}{\pi n\alpha \gamma_{n}(nkr)} \]

where
\[ \gamma_n(nkr) = \frac{K_n^{(nkr)}}{I_n(nkr)} \]

For an iron environmental screen
\[ D_n = -\beta_n(nkr)B_n[r_t, \alpha_t, \pm 1] \]
\[ \beta_n(nkr) = K_n^{(nkr)} \]

and for a conducting environmental screen
\[ D_n = -\gamma_n(nkr)B_n[r_t, \alpha_t, \pm 1] \]

and for no environmental screen
\[ D_n = 0 \]

Substitution of these constants with the magnetic-field expressions, eqns. 36 to 42, gives the magnetic field distributions for an individual phaseband listed in Table 1. Corresponding line current-density distributions are also shown in this Table. Magnetic-field distributions for a phasegroup, phase and total machine phases can now be obtained by progressive use of superposition: for the first stage, that of combining phasebands to form a phasegroup being shown in Table 3. Table 2 defines the environmental screen factors used in Tables 1 and 3.

11.3 Boundary conditions

(i) By comparison with the linear current density distributions of Section 11.1 for periodicity in \( \theta \) and \( z \)
\[ \nu = nk = \frac{nn}{2l} \]
the space harmonic number and
\[ B_0 = 0 \]
for a right-hand helical winding
\[ C_1 = 0, \quad C_2 = 1 \]
for a left-hand helical winding
\[ C_1 = 1, \quad C_2 = 0 \]

(ii) As \( r \to 0, K_n(nkr) \to \infty \) while the magnetic fields are finite.
\[ B_0 = 0 \]

(iii) For a helical winding in air (no environmental screen) as \( r \to \infty, I_n(nkr) \to \infty \) while the magnetic fields tend to zero. \( A_n = 0 \) for \( r > r_t \).

(iv) The boundary conditions at the environmental screen depend on the type of screen:

(a) No environmental screen. In this case no image winding is necessary and the previous boundary conditions are sufficient.

(b) Iron environmental screen, \( (\mu_\perp = \infty, \rho = \infty) \). All the flux enters the iron core normally such that at \( r = r_t \), \( H_\theta = H_z = 0 \).

(c) Conducting environmental screen \( (\mu_\perp = 1, \rho = 0) \). All the flux is excluded from the environmental screen such that at \( r = r_t \), \( H_\theta = H_z = 0 \) for all \( \theta \) and \( z \).

(d) At the stator current sheet radial flux density is continuous and
\[ H_{\text{outside}} - H_{\text{inside}} = K_{1n} \]
\[ H_{\text{outside}} - H_{\text{inside}} = -K_{2n} \]

11.4 Inductance calculations

11.4.1 Phasegroup inductance: The total flux entering the Gaussian surface of Fig. 5 from a phasegroup of \( Z/b \) turns centred at \( \theta = \alpha \) is
\[ \phi_{gn}(\alpha) = \mu_0 \int_{-z}^{+z} \int_{-z}^{+z} -H_{gn}[r, \alpha]rdrdz + \mu_0 \int_{-\pi}^{+\pi} \int_{-\pi}^{+\pi} H_{gn}[r, \alpha]rd\theta dr \]
Evaluating this integral and replacing the first helical loop by a phasegroup of Z/b turns and breadth factor \( k_n \) gives the mutual flux linkage between phasegroups as

\[
\Phi_{\alpha}(\alpha) = \frac{\mu_0 Z^2 a_n^2}{2 b h} i_{ph} \cos \alpha
\]

\[
\Phi_{\beta}(\alpha) = \frac{2 \mu_0 Z^2 b_n^2 k_n^2 \pi n_a i_{ph} \cos \alpha}{n b_r^2}
\]

where \( k_n \) is defined as the skew factor, and

\[
k_n = \frac{\gamma_n(n k_r)}{[K_n(n k_r) - \gamma_n(n k_r)]}
\]

\( k_n \) is the radial flux environmental screen factor defined in Table 2, column 1. Mutual inductance is obtained from

\[
M_{\alpha}(\alpha) = \frac{\Phi_{\alpha}(\alpha)}{i_{ph} a}
\]

giving

\[
M_{\alpha}(\alpha) = \frac{\mu_0 Z^2 a_n^2}{2 b} \cos \alpha
\]

\[
M_{\beta}(\alpha) = \frac{2 \mu_0 Z^2 b_n^2 k_n^2 \pi n_a \cos \alpha}{n b_r^2}
\]

for all \( n \)

the total mutual inductance being the sum of the harmonic components

\[
M(\alpha) = M_{\alpha}(\alpha) + \sum_{n=1}^{\infty} M_{\alpha}(\alpha)
\]

11.4.2 Phase inductance: Phasegroups may be interconnected in a variety of ways to produce a phase, the most usual being:

(i) a phase and phasegroup are identical

(ii) two phasegroups 180° phase displaced are connected in parallel

(iii) two phasegroups 180° phase displaced are connected in series to give

\[
M_{\alpha}(\alpha) = \frac{b_n^2}{4 a^2} M_{\alpha}(\alpha)
\]

\[
M_{\beta}(\alpha) = \frac{\mu_0 Z^2 b_n^2 k_n^2 \pi n_a \cos \alpha}{2 n a^2}
\]

\( n \) odd

11.4.3 Synchronous inductance: For an m-phase generator with no saliency the synchronous inductance is

\[
L_{s} = \frac{m}{2} M_{\alpha}(\alpha) = \frac{m n}{2}
\]

\( n = 2 \tau m \pm 1 \)

\( \tau \) any integer

giving

\[
L_{s} = \frac{m \mu_0 Z^2 b_n^2 k_n^2 \pi n_a \cos \alpha}{4 n a^2}
\]

11.4.4 Subtransient inductance: The image winding representing the outer-rotor screen introduces an additional magnetic scalar potential distribution of the form

\[
\Omega_{s} = F_{n} K_{n}(n k_r) \cos [n(\theta - k z) + \phi_{s}]
\]

r.h. helix

\[
\Omega_{s} = F_{n} K_{n}(n k_r) \cos [n(\theta + k z) + \phi_{s}]
\]

l.h. helix

which combined with eqns. 34 and 35 gives

\( (a) \) inside the winding

\[
\Omega_{s} = \{A_{n} + D_{n}\} K_{n}(n k_r) \cos [n(\theta - k z) + \phi_{s}]
\]

\( (b) \) outside the winding

\[
\Omega_{s} = \{D_{n} K_{n}(n k_r) + B_{n} + F_{n}\} K_{n}(n k_r) \cos [n(\theta + k z) + \phi_{s}]
\]

Introduction of the additional boundary condition at the outside radius of the outer rotor modifies the scalar magnetic potential and field distributions in such a way that the environmental screen factors \( k_m, k_p, k_n \) and \( k_m \) of Table 2 are modified to those of Table 4. Consequently, with an iron environmental screen the radial field outside the stator winding is given by

\[
H_{r} = \frac{K_{n}(n k_r)}{n a b} \frac{K_{n}(n k_r)}{n a b} \left[1 - \frac{1}{\gamma_{1}(n k_r)}\right] \sin n(\theta - a l + k z)
\]

For nonzero \( n \) there is no net axial flux entering the Gaussian surface of Fig. 5 and so the phase inductances are entirely due to radial magnetic fields and the subtransient inductance is

\[
L_{s} = \frac{m \mu_0 Z^2 b_n^2 k_n^2 \pi n_a \cos \alpha}{4 n a^2}
\]

143
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